MAT 162—Exam #3—4/24/15

Name: _____

Show all work using correct mathematical notation. Calculators are not permitted.

1. (10 points) Find the limit of the sequence

$$a_n = \frac{\ln(7n^3 + 8n + 1)}{\ln(5n^2 + 4)}.$$

Show your work using correct limit notation, and simplify your answer.

2. (10 points) Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{5}{2^{3n+1}}.$$

Give your answer in simplest possible form.

3. (10 points) Consider the sequence $a_n = 2 + e^{-n}$. Evaluate

(a) $\lim\limits_{n \to \infty} a_n$

(b) $\sum\limits_{n=1}^{\infty} a_n$

4. (20 points) Consider the series $S = \sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{\sqrt{n}}$.

(a) Determine whether the series converges absolutely, converges conditionally, or diverges. Justify your answer using appropriate tests.

(b) Write out the fifth partial sum, S_5.

(c) If we use the approximation $S \approx S_5$, what is the maximum possible error in our estimate?

5. (30 points) Decide whether each series converges or diverges, and justify your conclusions using appropriate tests. You must give coherent arguments to receive credit.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^8 n}{\sqrt{n^5 + 1}}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1/3}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{n!(n+1)!}$

6. (20 points) Find the radius and interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x+4)^n}{(2n+3)7^n}.$$

Justify your conclusions by citing appropriate tests.