MAT 161—Exam #2—6/10/15

Name: Solutions

Calculators are NOT allowed. Show all work using correct mathematical notation.

1. (25 points) Multiple choice. Circle the letter of the correct answer.

I. The instantaneous rate of change of the function $f(x) = \dfrac{x}{2x+1}$ at $x = 1$ is

(A) $\frac{1}{9}$ (B) $-\frac{1}{9}$ (C) $\frac{1}{3}$

(D) $-\frac{1}{3}$ (E) $\frac{1}{2}$ (F) $-\frac{1}{2}$

$$f'(x) = \frac{2x+1 - x \cdot 2}{(2x+1)^2}$$

$$= \frac{1}{(2x+1)^2}$$

II. The derivative of the function $f(x) = e^{\tan x}$ is $f'(x) =$

(A) $e^{\tan x}$ (B) $e^{\sec^2 x}$

(C) $e^{\tan x} \sec^2 x$ (D) $e^{\sec^2 x} \tan x$

(E) $(\tan x) e^{\tan x - 1}$ (F) $(\tan x) e^{\tan x - 1} \sec^2 x$

III. The slope of the tangent line to the curve $y = \ln x$ at the point $(e, 1)$ is

(A) 0 (B) 1 (C) -1 (D) e

(E) $\frac{1}{e}$ (F) $-e$ (G) $-\frac{1}{e}$ (H) undefined

$$\frac{dy}{dx} = \frac{1}{x}$$

IV. The derivative of the function $g(x) = x^2 \sin x$ is $g'(x) =$

(A) $2x \cos x$ (B) $x^2 \cos x + 2x \sin x$

(C) $3x^2 \sin(x^3)$ (D) $2x \sin x - x^2 \cos x$

(E) $2x^3 \sin x \cos x$ (F) $x^2 \cos x - 2x \sin x$

V. A particle's position in meters after t seconds is given by $s(t) = \cos(2t)$. Find the particle's acceleration (in m/s^2) at $t = \pi$.

(A) 0 (B) 1 (C) -1

(D) 2 (E) -2 (F) 4

(G) -4 (H) π (I) $-\pi$

$$v(t) = -2\sin(2t)$$

$$a(t) = -4\cos(2t)$$

2. (25 points) Calculate $\frac{dy}{dx}$ for each of the following functions. You do NOT need to simplify your answers.

(a) $y = 2^x \csc(3x)$

$$\frac{dy}{dx} = 2^x\left(-3\csc(3x)\cot(3x)\right) + \left(\csc(3x)\right)\cdot 2^x \ln 2$$

(b) $y = \dfrac{\sin^{-1} x}{x^3+7}$

$$\frac{dy}{dx} = \frac{(x^3+7)\cdot\frac{1}{\sqrt{1-x^2}} - (\sin^{-1}x)\cdot 3x^2}{(x^3+7)^2}$$

(c) $y = \sqrt{\ln(\ln x)}$

$$\frac{dy}{dx} = \frac{1}{2}\left(\ln(\ln x)\right)^{-1/2}\cdot\frac{1}{\ln x}\cdot\frac{1}{x}$$

(d) $y = \sin^9(e^{5x})$

$$\frac{dy}{dx} = 9\sin^8(e^{5x})\cdot\cos(e^{5x})\cdot 5e^{5x}$$

(e) $y = [\tan^{-1}(x \ln x)]^{10}$

$$\frac{dy}{dx} = \frac{10\left[\tan^{-1}(x\ln x)\right]^9}{1+(x\ln x)^2}\left(x\cdot\frac{1}{x} + \ln x\right)$$

3. (6 points) Find $h^{(10)}(0)$ for the function $h(x) = x^8 + e^{2x} + \sin x$.

$$h^{(10)}(x) = 2^{10} e^{2x} - \sin x$$

$$\Rightarrow \quad h^{(10)}(0) = 2^{10} = 1024$$

4. (7 points) Use logarithmic differentiation to find the derivative of the function $y = x^{2x}$.

$$\ln y = 2x \ln x$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = 2x \cdot \frac{1}{x} + (\ln x) \cdot 2$$

$$\Rightarrow \quad \frac{dy}{dx} = x^{2x}(2 + 2 \ln x)$$

5. (12 points) Find the equation of the tangent line to the curve $xy^3 + y^4 = 5$ at the point $(4, 1)$.

$$x \cdot 3y^2 \frac{dy}{dx} + y^3 + 4y^3 \frac{dy}{dx} = 0$$

$$\Rightarrow \quad (3xy^2 + 4y^3)\frac{dy}{dx} = -y^3$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y^3}{3xy^2 + 4y^3}$$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{(4,1)} = \frac{-1}{16}$$

6. (10 points) An object's position along a line is given by the formula $s(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t$, where s is measured in meters and t is measured in seconds. For what values of t is the particle moving in the negative direction?

$$v(t) = t^2 - 7t + 10$$

$$= (t-2)(t-5)$$

$+ \quad - \quad +$

$2 \quad 5$

$\therefore$ velocity is negative for $2 < t < 5$

7. (15 points) A triangle's base is decreasing at a constant rate of 2 cm/sec, and its area is increasing at a constant rate of 6 cm^2/sec. How fast is the triangle's height changing at the instant when the base is 10 cm and the height is 12 cm?

$$A = \frac{1}{2} bh$$

$$\Rightarrow \frac{dA}{dt} = \frac{1}{2}\left(b\frac{dh}{dt} + h\frac{db}{dt}\right)$$

At this instant,

$$6 = \frac{1}{2}\left(10\frac{dh}{dt} + 12 \cdot (-2)\right)$$

$$\Rightarrow 6 = 5\frac{dh}{dt} - 12$$

$$\Rightarrow \frac{dh}{dt} = \frac{18}{5} = 3.6$$

So the height is increasing at a rate of 3.6 cm/sec.