MAT 161—Exam #1—6/2/15

Name: Solutions

Calculators are NOT allowed. You must show all work using correct mathematical notation in order to receive credit.

1. (20 points) Evaluate each of the following limits using the methods discussed in class. Do not rely on numerical estimation or advanced techniques such as l'Hôpital's Rule.

(a) $\lim\limits_{x\to 5} \dfrac{x^2-25}{2x-10}$

$$= \lim_{x\to 5} \frac{(x-5)(x+5)}{2(x-5)}$$

$$= \lim_{x\to 5} \frac{x+5}{2}$$

$$= 5$$

(b) $\lim\limits_{x\to 0} \dfrac{\tan 3x}{4x}$

$$= \lim_{x\to 0} \frac{\sin 3x}{4x\cos 3x}$$

$$= \frac{3}{4} \lim_{x\to 0} \frac{\sin 3x}{3x} \lim_{x\to 0} \frac{1}{\cos 3x}$$

$$= \frac{3}{4}\cdot 1\cdot 1$$

$$= \frac{3}{4}$$

(c) $\lim\limits_{x\to 1^+} \dfrac{x+2}{x-1}$ " $\dfrac{3}{0^+}$ "

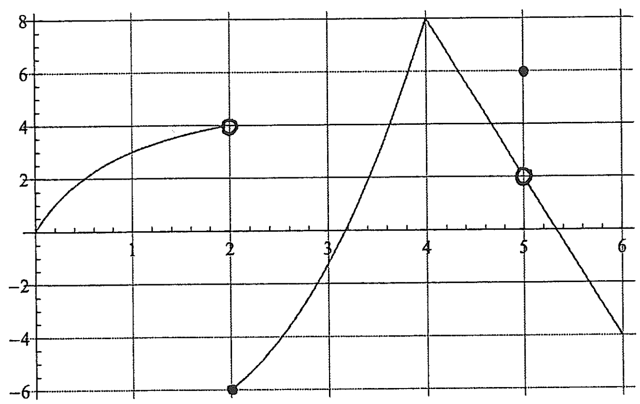
$$= \infty$$

(d) $\lim\limits_{x\to -\infty} \dfrac{3x^2+7x+10}{5x^3-x^2+6}$

$$= \lim_{x\to -\infty} \frac{\frac{3}{x}+\frac{7}{x^2}+\frac{10}{x^3}}{5-\frac{1}{x}+\frac{6}{x^3}}$$

$$= 0$$

2. (20 points) Consider the function $y = f(x)$ graphed below.



(a) Evaluate each of the following quantities. If a quantity does not exist, say so. Explanations are not required.

(i) $\lim_{x \to 2^-} f(x)$

$= 4$

(ii) $f(2)$

-6

(iii) $\lim_{x \to 4} f(x)$

$= 8$

(iv) $\lim_{x \to 5} f(x)$

$= 2$

(b) For what values of x does f fail to be continuous?

$x = 2, \quad x = 5$

(c) For what values of x does f fail to be differentiable?

$x = 2, \quad x = 4, \quad x = 5$

3. (20 points) Consider the function

$$f(x) = \begin{cases} x^3 - 1 & \text{if } x < 2 \\ 2x + 3 & \text{if } 2 \le x < 5\,. \\ x^2 & \text{if } x > 5 \end{cases}$$

(a) Evaluate each of the following, or state that it does not exist.

(i) $f(2)$

$= 7$

(ii) $\lim_{x \to 2^-} f(x)$

$= 7$

(iii) $\lim_{x \to 2^+} f(x)$

$= 7$

(iv) $\lim_{x \to 5^-} f(x)$

$= 13$

(v) $\lim_{x \to 5^+} f(x)$

$= 25$

(vi) $\lim_{x \to 5} f(x)$

does not exist

(b) For what values of x does f fail to be continuous? Justify your answer using the results of part (a).

$x = 5$ since $\lim_{x \to 5} f(x)$ does not exist

For the problems on this page, you may use shortcut methods for computing derivatives.

4. (10 points) Find the derivative of each function below.

(a) $f(x) = 2x^5 + 7e^x + 1$

$f'(x) = 10x^4 + 7e^x$

(b) $g(x) = \frac{3}{x} - 4x^\pi + e^2$

$g'(x) = -\frac{3}{x^2} - 4\pi x^{\pi - 1}$

5. (10 points) An object's position along a straight line is given by $s(t) = t^2 + t$, where s is measured in feet and t is measured in seconds.

(a) Calculate the object's average velocity over the interval $[1, 4]$.

$\frac{s(4) - s(1)}{4 - 1} = \frac{20 - 2}{3} = 6$ ft/sec

(b) Calculate the object's instantaneous velocity at $t = 1$.

$s'(t) = 2t + 1$

$\Rightarrow s'(1) = 3$ ft/sec

6. (20 points) Consider the function $f(x) = \sqrt{x}$.

(a) Use the **definition of the derivative in terms of a limit** to calculate $f'(4)$. No credit will be given for shortcut methods, and you must use correct limit notation in your solution.

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2}$$

$$= \lim_{h \to 0} \frac{4+h-4}{h(\sqrt{4+h}+2)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{4+h}+2}$$

$$= \frac{1}{4}$$

(b) Find the equation of the tangent line to the curve at the point $(4, 2)$. If you could not do part (a), you may apply shortcut methods to do this part of the problem.

$$y - 2 = \frac{1}{4}(x - 4)$$

$$\Rightarrow \quad y = \frac{1}{4}x + 1$$