McKibben Webster – Chapter 7 Partial Solutions and Hints

7.2.1 (i) $x = 0$ is one solution. Find another using separation of variables.

(ii)(a) This follows directly by using separation of variables.

7.4.1 $\lim_{n\to\infty} \frac{3\left(\frac{1}{n}\right)^{5/8}}{\frac{1}{n}} = \infty$

7.4.2 This condition when M_g is constant means the chord line slopes are uniformly bounded. If M_g depends on t, then the condition is different and is "local" in nature.

7.4.3 (i) Use $\delta = \frac{\varepsilon}{M_f + 1}$ in the definition of continuity. Tell how.

(ii) Take $f(x) = \sqrt{x}$ on [0,1].

(iii) Use the Mean Value Theorem in the proof.

(iv) Use $f(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & 0 < x \le 1 \\ 0, & x = 0 \end{cases}$

7.4.4 Yes. Use $\left\| \sum_{i=1}^{n} \left(f_i(x) - f_i(y) \right) \right\| \le \sum_{i=1}^{n} \left\| f_i(x) - f_i(y) \right\|$. How?

7.4.5 (i) This follows because

$$\begin{aligned} |f(x) - f(y)| &= |x-y||x+y| \\ &\le |x-y|\left[|x|+|y|\right] \\ &\le |x-y| \cdot \max\{|a|, |b|\} \end{aligned}$$

(ii) No. The chord line slopes are not uniformly bounded. A sequence of intervals can be formed for which the absolute values of the chord line slopes become infinite.

7.5.1 Because $M = 0$ (in the lemma itself) in this application.