McKibben Webster – Chapter 2 Partial Solutions and Hints (Part 2)

2.6.1 These follow from the corresponding properties of real numbers.

2.6.2 (i) Circle centered at (0,0) with radius ε

(ii) Points outside a sphere centered at (1,0,0) with radius ε

(iii) $\left\{\overrightarrow{x_0}\right\}$

2.6.3 (i) Use the triangle inequality for the first inequality and Young's inequality for the second.

(ii) The strategy is the same, but with more terms.

2.6.4 (i) – (v) follow directly from the definition.

(vi) The hypothesis implies that $\langle \mathbf{x}-\mathbf{y},\mathbf{z}\rangle = 0,\ \forall \mathbf{z}\in\mathbb{R}^N$. Choose $\mathbf{z}=\mathbf{x}-\mathbf{y}$ and calculate. Now what?

2.6.5 A circle or sphere centered at $\mathbf{x}_0$ with radius ε .

2.6.6 (i), (iii), (iv) Yes (ii) No

2.6.7 (i) a, b

(ii) $A^{-1}=\begin{bmatrix} a^{-1} & 0 \\ 0 & b^{-1}\end{bmatrix}$; a^{-1}, b^{-1}

(iii) λ^{-1}

2.7.1 (i) As $0<|t-a|\to 0,\ \|f(t)-L\|_{\mathbb{R}^N}\to 0,$ where $L\in\mathbb{R}^N$.

(ii) As $0<|t-a|\to 0,\ \|F(t)-L\|_{M^N}\to 0,$ where $L\in M^N$.

2.7.2 (i) <2, 0, -1>

(ii) $\begin{bmatrix} 1 & 0 \\ 2 & 1\end{bmatrix}$

(iii) DNE

2.7.3 (i) As $0<|t-a|\to 0,\ \|f(t)-f(a)\|_{\mathbb{R}^N}\to 0$

(ii) As $0<|t-a|\to 0,\ \|F(t)-F(a)\|_{M^N}\to 0$

(iii) $h(\mathbb{R}) \subset domF$ and *h*, *F* are both continuous on their domains

2.7.4 (i) $\lim_{t\to a} G(t)$, provided this limit exists

(ii) Similar to (i) – use the product rule for limits

(iii) Same as (i) because the mapping $t \mapsto \frac{t}{\beta}$ is a continuous mapping. Tell why.

2.7.5 For every *M* > 0, show there exists $\delta > 0$ for which $h \in (-\delta, \delta)$ and $\left|\frac{f(c+h)-f(c)}{h}\right| \geq M$.

2.7.6 (i) and (ii) Invoke Exercise 2.7.1. Tell how and why you can use this.

2.7.7 $(\alpha F)' = \alpha F'$. Use the result of Exercise 2.7.6 to prove this.

2.7.8 (i) Use Exercise 2.7.7

(ii) Think "product rule"

(iii) Think "product rule" and chain rule to differentiate $t \mapsto h\left(\frac{t}{\beta}\right)$.

2.7.9 Yes. You can use the known result for real-valued functions and apply it componentwise using Exercise 2.7.6 (i). Tell how carefully.

2.7.10 (i) and (ii) Since the integral is a limit, invoke Exercise 2.7.1. Tell how carefully.

2.7.11 $\begin{bmatrix} A & B & O \\ B & A & C \\ 0 & C & A \end{bmatrix}$, where $A = e^t\left(-te^{-t} - e^{-t} + 1\right), B = e^t\left(-e^{-t} + 1\right), C = e^t\left(-1 + e^{-t}\right)$.

2.7.12 (i) Limit of a sequence of vectors is computed componentwise

(ii) Same as (i)

(iii) Componentwise computations work in $\mathbb{R}^N$

2.7.13 Mimic Exercise 2.5.7 (i). Tell how.

2.7.14 The series is geometric given by $\sum_{m=p}^{\infty}\left(\frac{1}{|a|^2}\right)^p$. So what?

2.7.15 The series is geometric given by $\sum_{m=p}^{\infty}\left(\frac{1}{6}\right)^p$. So what?

2.7.16 $\|f(t)\| \leq \sqrt{a^2 + b^2 + c^2}$, for every *t*. Why? So what?

2.7.17 The norm of both components is less than or equal to 1, for every t. Why? So what?

2.7.18 Yes.

2.7.19 (i) They must go to zero.

(ii) The diagonal entries go to1 and all other entries go to zero.

(iii) Limits are computed componentwise. So what?

2.7.20 (i) $\|A_m x - Ax\| = \|(A_m - A)x\|$. So what?

(ii) $\|Ax_m - Ax\| \le \|A\|\|x_m - x\|$. So what?

(iii) $\|A_m x_m - Ax\| = \|A_m x_m - Ax_m + Ax_m - Ax\| \le \|A_m - A\|\|x_m\| + \|A\|\|x_m - x\|$. Now use (i) and (ii).

2.8.1 (i) Separate the variables as $\sin(\pi y)dy = e^{2x}dx$.

(ii) Simply integrate both sides with respect to x.

(iii) $\left(1 - y^3\right)dy = \sum_{i=1}^{N} a_i \sin\left(b_i x\right)dx$. Now continue…

2.8.3 The equation is linear – use (2.44) directly.

2.8.4 (i) m_1 and m_2 must have negative real parts. Why?

(ii) The real parts of m_1 and m_2 are less than or equal to zero. Why?